

Systematics of 2_1^+ states in semi-magic nuclei

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We propose a simple systematics of low lying 2_1^+ energy levels and electromagnetic transitions in semi-magic isotopic chains $Z = 28, 50, 82$ and isotonic chains $N = 28, 50, 82, 126$. To this purpose we use a two-level pairing plus quadrupole Hamiltonian, within the spherical Quasiparticle Random Phase Approximation (QRPA). We derive a simple relation connecting the 2_1^+ energy with the pairing gap and quadrupole-quadrupole (QQ) interaction strength. It turns out that the systematics of energy levels and $B(E2)$ values predicted by this simple model is fulfilled with a reasonable accuracy by all available experimental data. Both systematics suggest that not only active nucleons but also those filling closed shells play an important role.

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Semi-magic nuclei are important for nuclear structure studies, due to the fact that relative simple shell-model configurations are used in describing collective states. The 2_1^+ energy in semi-magic nuclei within the generalized seniority scheme is constant along a given isotopic/isotonic chain [1]. Talmi's classical example is Sn isotopic chain where the 2_1^+ energy is fairly constant throughout the complete shell from $N=52$ up to $N=80$ [2]. However, the 2_1^+ energy in other semi-magic nuclei is not a constant and depends on the matrix elements of the effective interaction.

There are several systematic studies concerning spectroscopic properties of even-even nuclei [3–8]. In this paper we will investigate low lying 2_1^+ states in semi-magic isotopic and isotonic chains, by using a spherically symmetric Hamiltonian with two levels, one for protons and one for neutrons. In our calculations we have used the experimental binding energies [9] in order to estimate the pairing gaps, which have nonvanishing values for magic proton or neutron numbers. Thus, we include both kinds of nucleons because not only active particles but also the particles filling the closed shell are important in describing low lying collective states. The Hamiltonian contains the most relevant degrees of freedom, given by the pairing and quadrupole-quadrupole (QQ) two body interactions. By using the standard quasiparticle representation one has

$$H = E_p N_p + E_n N_n - \frac{1}{2} (\kappa_p Q_p Q_p^\dagger + \kappa_n Q_n Q_n^\dagger) + \kappa_{pn} Q_p Q_n^\dagger + \kappa_{pn} Q_n Q_p^\dagger, \quad (1)$$

where E_τ ($\tau = p, n$) denotes the quasiparticle energy, N_τ is the number of particles operator and

$$Q_\tau = \xi_\tau [A_{2\mu}^\dagger(\tau) + (-)^\mu A_{2-\mu}(\tau)], \quad (2)$$

is the quadrupole operator written in terms of the normalized two quasiparticle excitation operator $A_{2\mu}^\dagger(\tau)$, defined in a standard way by the angular momentum

coupling of two quasiparticle creation operators of the same kind. We describe low lying 2^+ excitations in terms of the QRPA phonon operator

$$\Gamma_{2\mu}^\dagger = \sum_{\tau=p,n} [X_\tau A_{2\mu}^\dagger(\tau) - Y_\tau (-)^\mu A_{2-\mu}(\tau)] . \quad (3)$$

The QRPA equation of motion $[H, \Gamma_{2\mu}^\dagger] = E_{2+} \Gamma_{2\mu}^\dagger$ leads to the standard system of equations [10]. By introducing the following short hand notations

$$x_\tau = \kappa_\tau \xi_\tau \xi_\tau, \quad \tau = p, n; \quad x_{pn} = \kappa_{pn} \xi_p \xi_n, \quad (4)$$

the QRPA roots can be written as follows

$$E_{2+}^2 = 2[E_p^2 + E_n^2 - E_p x_p - E_n x_n] \pm \sqrt{(E_p^2 - E_n^2 - E_p x_p + E_n x_n)^2 + 4E_p E_n x_{pn}^2} . \quad (5)$$

In the absence of the proton-neutron interaction, i.e. $x_{pn} = 0$, one obtains two roots for neutron and proton systems separately

$$E_{2+}^2 = 4[E_\tau^2 - E_\tau x_\tau], \quad \tau = n, p . \quad (6)$$

By using the expression of the quasiparticle energy in terms of the Lagrange multiplier (we set the single particle energy to zero) and pairing gap, $E_\tau = \sqrt{\lambda_\tau^2 + \Delta_\tau^2}$, one obtains the following simple relation

$$(E_{2+}/2)^2 - \Delta_\tau^2 = \lambda_\tau^2 - y_\tau \Delta_\tau, \quad (7)$$

where the coefficient is given by

$$y_\tau = x_\tau f(z_\tau) . \quad (8)$$

Here, we introduced the following universal function

$$f(z_\tau) = \sqrt{1 + (\lambda_\tau/\Delta_\tau)^2} = \frac{1}{\sqrt{z_\tau(2 - z_\tau)}} \quad (9)$$

$$z_\tau \equiv N_\tau/\Omega_\tau ,$$

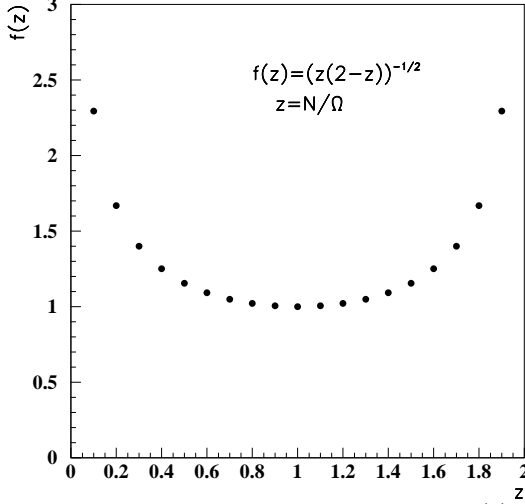


FIG. 1: The function defined by Eq. (9)

where we used the number of particles condition in terms of the valence particle number N_τ . This function is plotted in figure 1. One sees that around the middle of the shell ($N_\tau = \Omega_\tau$) this function has an almost constant value $f(z) \approx 1$. Actually for the regions of a shell where $f(z)$ varies a lot (that is near the beginning and end) the 2^+ states are not very collective so they are outside our perspective. Eq. (6) can be rewritten as follows

$$E_{2^+}^2 = 4[\bar{\Delta}_\tau^2 - \bar{\Delta}_\tau x_\tau], \quad \bar{\Delta}_\tau \equiv \Delta_\tau f(z_\tau). \quad (10)$$

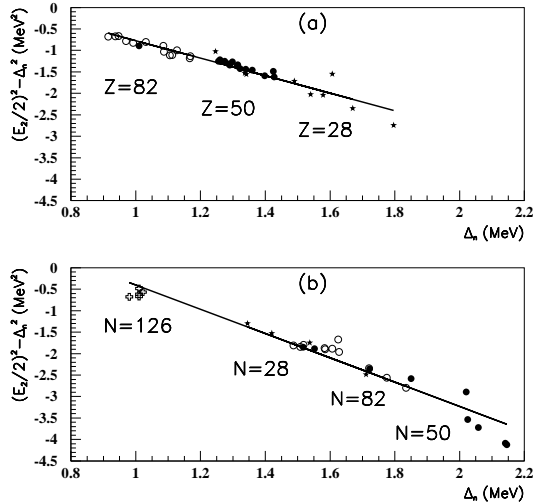


FIG. 2: (a) Fit of equation (7) for the neutron-like root for $Z=28$ (stars), 50 (dark circles), 82 (open circles) isotopic chains. (b) Same as in (a), but for $N=28$ (stars), 50 (dark circles), 82 (open circles), 126 (open crosses) isotonic chains.

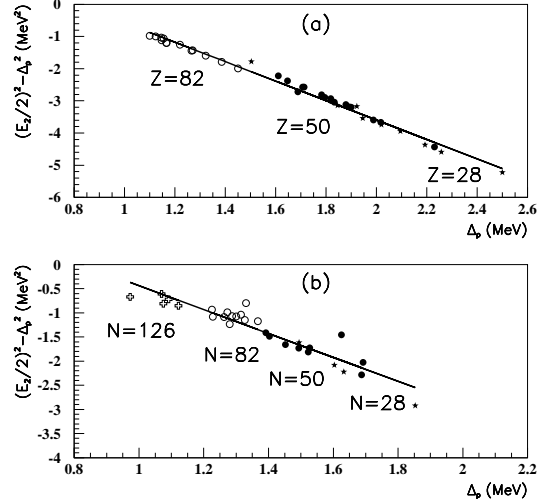


FIG. 3: Same as in figure 2, but for the proton-like root.

The fitting procedure of the general relation (5) showed that the proton-neutron interaction x_{pn} has vanishing values along various isotopic and isotonic chains, except for the $Z=28$ isotopic chain, because the proton-neutron interaction plays an important role for $N \approx Z$ nuclei.

Concerning the parameter y_τ , we considered two versions. In the first approximation we supposed a constant value of y_τ in Eq. (7). Later on, we will consider a mass dependent parameter.

In Ref. [11] the averaged two-body matrix elements in odd-odd nuclei extracted from experimental data versus the angle between the orbital angular momenta of the interacting particles were analyzed. As an interesting observation, it turns out that in most cases the shape of this dependence is proportional to the function $1/f(z_\tau)$. Thus, in principle the function $f(z_\tau)$ could be compensated by the variation of the parameter x_τ , keeping in this way the parameter y_τ in Eq. (8) almost constant.

In figure 2 (a) one sees how the relation (7) is nicely fulfilled for the neutron-like root along various isotopic chains (even for $Z=28$). It is important to point out that the slope parameter y_τ has indeed an almost constant value.

Table I
Fitting parameters in Eq. (7) with $\tau = n, p$ for isotopic (a) and isotonic (b) chains

| Figure | τ | λ_τ^2 | y_τ | σ |
|--------|--------|------------------|----------|----------|
| 2 (a) | n | 1.267 | 2.040 | 0.128 |
| 2 (b) | n | 2.434 | 2.832 | 0.228 |
| 3 (a) | p | 2.451 | 3.023 | 0.118 |
| 3 (b) | p | 2.038 | 2.475 | 0.188 |

An interesting fact is revealed by figure 2 (b), showing that the same relation is also fulfilled by the iso-

tonic chains, i.e., by those neutrons filling the corresponding closed shells. Thus, the pairing effects of these neutrons are also important in explaining the low lying 2_1^+ collective state.

The straight lines in figure 2 are defined by the fitting parameters λ_τ^2 and y_τ , given in the Table I for isotopic (a) and isotonic (b) chains.

It is very important to stress on the fact that the same relation is also fulfilled for the corresponding proton-like root, as can be seen from figure 3. Thus, as in the previous case, for the isotopic chains (a) the proton pairing effects play an important role. It is interesting to point out that the best fit is given in figure 3 (a), i.e., by protons filling a closed shell.

The next step is to consider a variable parameter y_τ , by using Eq. (10). In figure 4 (a) we plotted by dark circles the parameter x_n for various isotopic chains determined by this equation *by considering* $f(z_\tau) = 1$. In figure 4 (b) the parameter x_p for various isotonic chains is given by dark circles. One sees that the fitting curve given by

$$y_\tau = x_\tau = 5.2 A^{-0.35}, \quad (11)$$

approximates in a satisfactory way the computed values. We stress the fact that Eq. (10) with $f(z) = 1$ is a particular case of Eq. (7) with $\lambda_\tau = 0$, but with a *variable parameter* y_τ . Let us mention here that in Ref. [12] a much stronger dependence was derived, but only for the coupling strength $\kappa_\tau \sim A^{-2.2}$.

On the other hand, our analysis revealed that the values of the parameter x_p , but for *closed shell isotopic chains*, and x_n for *closed shell isotonic chains* have much more scattered values than in figure 4. This means that the "natural" values of the parameter x_p *along open shell isotonic chains* and x_n *along open shell isotonic chains* are more appropriate to be used in our systematics.

By using the above scalling law (11) we plotted by open circles in figure 5 (a) the energies computed according to Eq. (10) with $\tau = n$ for various isotopic chains. In figure 5 (b) similar values with $\tau = p$ for various isotonic chains are given. One remarks that, except for $Z = 28$ isotopes and $N = 28$ isotones, the experimental values given by dark circles are satisfactorily reproduced.

The fact that both proton and neutron roots are important is also supported by an analysis of electromagnetic transitions. The E2 transition matrix element, connecting the excited state $|k\rangle$ with the ground state $|0\rangle$, is given by the following standard expression

$$\begin{aligned} \langle 0 || T_2 || k \rangle &= \sum_{\tau=p,n} e_\tau \xi_\tau \left[X_\tau^{(k)} + Y_\tau^{(k)} \right] \\ &= \sum_{\tau=p,n} e_\tau \xi_\tau \sqrt{\frac{2E_\tau}{E_{2+}}}, \end{aligned} \quad (12)$$

where e_τ denote the effective proton and neutron charges. We compared experimental values with the quantity $B(E2 : k \rightarrow 0) = |\langle 0 || T_2 || k \rangle|^2$.

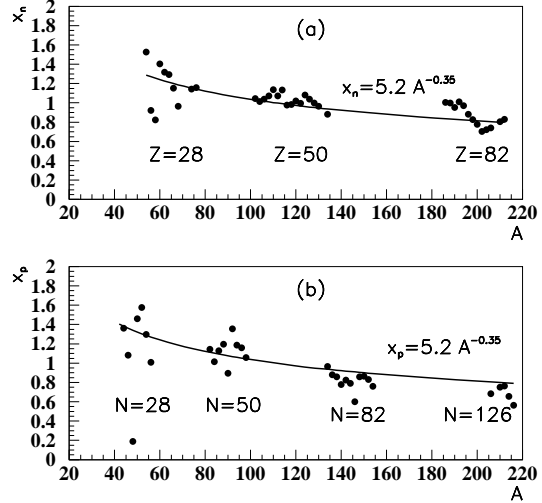


FIG. 4: The parameter x_n determined by Eq. (10) with $f(z_\tau) = 1$ for isotopic chains (a) and x_p for isotonic chains (b). The curve is given by the common Eq. (11).

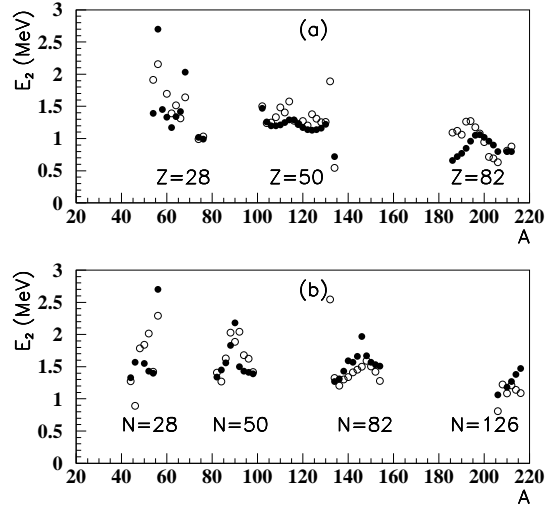


FIG. 5: Open circles denote the energies given by Eq. (10) with $\tau = n$ for various isotopic chains (a) and with $\tau = p$ for various isotonic chains (b). Experimental values are given by dark circles.

In figure 6 we plotted $B(E2)$ values for various isotopic (a) and isotonic chains (b). By dark circles experimental values are given, while open circles denote the fit with respect to the parameters $a_\tau = \sqrt{2}e_\tau\xi_\tau$. In Table II we give the fitting parameters a_τ for these chains. Indeed, one remarks that both proton and neutron effective charges have simultaneously nonvanishing and comparable values with opposite signs, i.e., the pairing effects are very strong for both kinds of particles.

In conclusion, we have analyzed low lying collec-

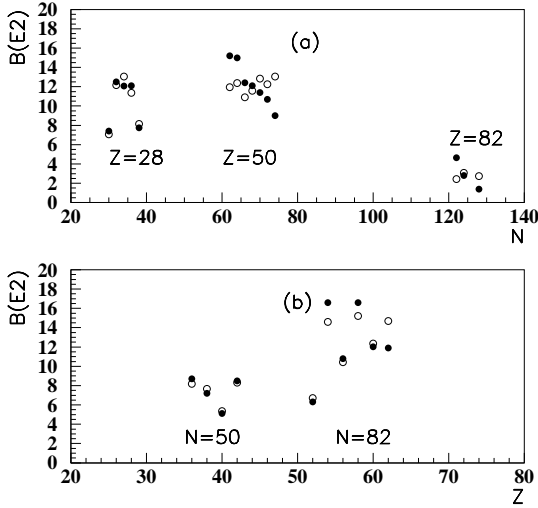


FIG. 6: (a) $B(E2)$ values versus neutron number for various isotopic chains. Experimental values are given by dark circles, while open circles denote the fit (12) with respect to parameters $a_\tau = \sqrt{2}e_\tau\xi_\tau$. (b) Same as in (a), but for various isotonic chains.

tive 2_1^+ states in semi-magic nuclei, where experimental data concerning energy levels and $B(E2)$ values are available. We used a toy Hamiltonian with one proton and one neutron level and a pairing plus quadrupole force as the two body interaction. The collective 2_1^+ state we described within the spherical QRPA. It turns out that, except for the $Z=28$ isotopic

chain, the proton-neutron QQ interaction has vanishing values. A very simple relation connecting the $E_{2_1^+}$ energy with the pairing gap and interaction parameter x_τ allows us to describe with a reasonable accuracy experimental data. This law is fulfilled with a good accuracy for both proton and neutron roots. This feature clearly shows the important role played by "inert" particles inside the closed shells. This conclusion is also supported by the analysis of $B(E2)$ values. Indeed, both proton and neutron effective charges with opposite signs should have nonvanishing values in order to explain the experimental data.

Table II

Fitting parameters $a_\tau = \sqrt{2}e_\tau\xi_\tau$ in Eq. (12) with $\tau = n, p$ for isotopic chains in figure 6 (a) and isotonic chains in figure 6 (b).

| Chain | a_p | a_n | σ |
|-------|--------|---------|----------|
| Z=28 | 9.998 | -6.185 | 0.041 |
| Z=50 | 7.036 | -3.499 | 0.131 |
| Z=82 | -2.169 | 2.950 | 0.253 |
| N=50 | -4.742 | 7.473 | 0.034 |
| N=82 | 81.627 | -84.446 | 0.084 |

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